Q.P. Code – 22633

Sixth Semester B.Sc. Degree Examination, September 2020

(Semester Scheme)

MATHEMATICS

Paper VII

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

Smt Siddaganga College of Arts
Science & Commerce for women
LIBRARY, TUMKUR.

1. Define a Vector Space.

2. State the necessary and sufficient conditions for a non-empty subset W of $V(F)$ to be a subspace of $V(F)$.

3. Show that $S = \{(1, 3, 2), (1, -7, -8), (2, 1, -1)\}$ is linearly dependent.

4. Verify whether the set $S = \{(2, 1), (1, -2), (1, 0)\}$ is a basis of R^2.

5. Define Range space of a Linear Transformation.

6. Show that, $T : R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (a_2, a_3)$ is a Linear Transformation.

7. If $T : U \to V$ is a Linear Transformation, prove that $T(0) = 0'$.

8. Evaluate $\int_C xdy - ydx$, along the line $y = x$ from (0, 0) to (1, 1).

9. Evaluate $\int_0^1 \int_1^2 (x^2 + y^2) dy\, dx$.

10. Evaluate $\iint_R \frac{1}{x^2 + y^2} dx\, dy$ where R is the region $x^2 + y^2 = a^2$.

11. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dx\,dy\,dz$.

12. Evaluate $\int_0^{\pi/2}\int_0^{a\cos\theta}\int_0^1 z\,dz\,dr\,d\theta$.

13. Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}=(x^2-y^2)\hat{i}+xy\,\hat{j}$ and 'C' is $y=x^3$ from (0, 0) to (2, 8).

14. Show that the area bounded by a simple closed curve 'C' is given by $\frac{1}{2}\int_C x\,dy-y\,dx$ by using Green's Theorem.

15. Using Gauss divergence theorem, find $\iint_S \vec{F}\cdot\hat{n}\,dS$ when $\vec{F}=ax\,\hat{i}+by\,\hat{j}+cz\,\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$.

16. State Stokes Theorem.

17. If $\vec{F}=\nabla\times\vec{A}$, show that $\iint_S \vec{F}\cdot\hat{n}\,dS=0$.

18. Define variational problem of Calculus of variation.

19. Find the Euler's equation for a functional $I=\int_{x_1}^{x_2} y'\left[\left(1+x^2y'\right)\right]dx$.

20. Solve : $I=\int_{x_1}^{x_2}\left(y^2+x^2y'\right)dx$.

II. Answer any **FOUR** of the following : **(4 × 5 = 20)**

1. Prove that A non-empty subset W of a vector space V is a subspace of V iff V $\alpha,\beta\in W$ and $C_1, C_2\in F\Rightarrow C_1\alpha+C_2\beta\in W$.

2. Express (3, –7, 6) as a Linear combination of the vectors (1, –3, 2), (2, 4, 1), (1, 1, 1).

3. Find the basis and dimension of the subspace spanned by the vector (1, 2, 3), (3, 1, 0), (–2, 1, 3).

4. Find the Linear Transformation, $T: R^2 \to R^2$, $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$.

5. State and prove Rank-nullity Theorem.

6. Verify Rank-nullity Theorem for the Linear Transformation $T: R^2 \to R^3$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Evaluate $\int_C (3x - 2y)dx + (y + 2z)dy - x^2 dz$ when 'C' is the curve $x = t, y = 2t^2$, $z = 3t^3$ and $0 \le t \le 1$.

2. Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) dy\, dx$.

Sree Siddaganga College of Arts
Science & Commerce for women
LIBRARY, TUMKUR

3. Evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$ by changing the order of Integration.

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$.

5. Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0, y = 0$, $z = 0$ and $y = b$.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Evaluate $\int_C (x + 2y)dx + (4 - 2x)dy$ around the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the counter clockwise direction.

2. Evaluate $\iiint_V div\, \vec{F}$, when $\vec{F} = x^2\hat{i} - y^2\hat{j} + 3yz\hat{k}$ and v is the volume bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2, z = 0$.

Q.P. Code – 22633

3. State and prove Green's theorem in a plane.

4. Evaluate $\iint\limits_S \vec{F}\cdot\hat{n}\,dS$ using divergence Theorem when $\vec{F} = x\hat{i} - y\hat{j} + (z^2-1)\hat{k}$ and S is the closed surface bounded by the places $z=0, z=1$ and the cylinder $x^2+y^2=4$.

5. Evaluate $\oint\limits_C \vec{F}\cdot d\vec{r}$ by Stokes Theorem when $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and 'C' is the boundary of the Triangle with vertices as (0, 0), (1, 0) and (1, 1).

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the extremal of the functional $I = \int_0^{\frac{\pi}{2}}\left[y^2 - (y')^2 - 2y\sin x\right]dx$ under the end conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$.

2. Prove that the Geodesic on a right circular cylinder is a circular helix.

3. Show that the curve which when rotated about a line generates a surface of minimum area is a catenary.

4. Find the extremal of the Functional $I = \int_0^{\pi}\left[(y')^2 - y^2\right]dx$ under the conditions $y(0)=0, y(\pi)=1$ and subjected to the constraint $\int_0^{\pi} y\,dx = 1$.
