Q.P. Code – 42634

Sixth Semester B.Sc. Degree Examination, September 2020

(CBCS Scheme)

Mathematics

Paper 6.2(a) – NUMBER THEORY

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

PART – A

I. Answer any **SIX** of the following : **($6 \times 2 = 12$)**

1. If a/b and 'x' is any integer then prove that a/bx.

2. Find whether the number 853 is prime or not.

3. Define the Linear Diophantine equation. Give an example.

4. If $a \equiv b(\bmod m)$ and 'n' is any positive integer, then prove that $a^n \equiv b^n(\bmod m)$.

5. Define pseudo prime. Give an example.

6. Find the highest power of 3 contained in 40!.

7. Evaluate τ and σ for $n = 3000$.

PART – B

II. Answer any **SIX** of the following : **($6 \times 3 = 18$)**

8. If $(a, b) = 1$ and $(a, c) = 1$, then prove that $(a, bc) = 1$.

9. If 'a' and 'c' are relatively prime and a/bc, then prove that a/b.

10. Prove that $3^{2n} + 7$ is a multiple of 8.

11. Find the digit in the unit place of the number 3^{127}.

12. Solve $36x + 12 \equiv 0 \pmod{15}$.

13. For any positive integer 'n', prove that $\phi(n) = n - 1$ if and only if 'n' is a prime.

14. Prove that the sum of the positive integers less than n and relatively prime to 'n' is equal to $\frac{1}{2}n\phi(n)$.

PART – C

III. Answer any **FOUR** of the following : **(4 × 5 = 20)**

15. Find (506, 1155) and express it in the form 506 m + 1155 n.

16. Prove that the smallest divisor (>1) of an integer (>1) is always a prime number.

17. If 'n' is even, then prove that $n(n+1)(n+2)$ is divisible by 24.

18. Find the general solution of $170x + 455y = 625$.

19. State and prove the Fundamental theorem of Arithmetic.

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. Prove that the relation "Congruence modulo m" is an equivalence relation on the set of integers.

21. Find the remainder when 3^{287} is divided by 23.

22. Find all 'x' which simultaneously satisfy the congruences,

$$x \equiv 1 \pmod 3$$
$$x \equiv 2 \pmod 4$$
$$x \equiv 3 \pmod 5$$

23. State and prove Fermat's Little theorem.

24. If p is a prime, then show that $q(p-3)! + 1$ is divisible by p.

V. Answer any **FOUR** of the following : **(4 × 5 = 20)**

25. Let 'm' and 'n' be any two positive integers, then prove that the arithmetic functions τ and σ are multiplicative.

26. If $F(n)$ is a multiplicative function and $F(n) = \sum_{d/n} f(d)$, then prove that 'f' is also multiplicative.

27. Prove that the highest power of a prime 'P' which divides $n!$ is $\sum_{i=1}^{K}\left[\frac{n}{P}\right]$, where $P^K \leq K \leq P^{K+1}$.

28. If $n = P_1^{K_1} \cdot P_2^{K_2} \cdots P_r^{K_r}$ is the canonical representation, then prove that

$$\phi(n) = n\left(1 - \frac{1}{P_1}\right)\left(1 - \frac{1}{P_2}\right)\cdots\left(1 - \frac{1}{P_r}\right)$$

29. State and prove Euler's generalisation of Fermat's theorem.