Q.P. Code – 42633

Sixth Semester B.Sc. Degree Examination, September 2020

(CBCS Scheme)

Mathematics

Paper 6.1 – COMPLEX ANALYSIS AND NUMERICAL METHODS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions. Answers should be in English only.

PART – A

I. Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Find the derivative of $f(z) = z^2$ by using Cauchy-Reimann equation.

2. Show that $u = x^3 - 3xy^2$ is a harmonic function.

3. Find the radius of convergence of the series $\sum_{n=0}^{\infty} n^n \cdot z^n$.

4. State Cauchy's integral theorem.

5. Evaluate : $\int_0^{3+i} z^2 dz$ along the line $3y = x$.

6. Evaluate : $\Delta(\log 2x)$.

7. Show that $\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$.

PART – B

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

8. Show that $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ represents a line passing through the origin.

9. Find the Analytic function whose imaginary part is $\cos x \cdot \cosh y$.

10. Show that $f(z) = \log z$ is analytic function.

11. Evaluate : $\int_C \frac{e^{2z}}{(z-2)^2}$ where $C : |z| = 3$.

12. State and prove Fundamental theorem of algebra.

13. Evaluate : $\int_0^4 y\,dx$ using Trapezoidal for the following values :

x :	0	1	2	3	4
y :	0.146	0.161	0.176	0.190	0.204

14. If $u_0 = 148,\ u_1 = 192,\ u_2 = 241,\ u_4 = 274$ find u_3.

PART – C

III. Answer any **FOUR** of the following : **(4 × 5 = 20)**

15. Show that $\arg\left(\frac{z+1}{z-1}\right) = \frac{\pi}{4}$ is a circle and find its centre, radius.

16. Show that $f(z) = \cosh z$ is analytic and also find its derivative.

17. If (r, θ) are the polar co-ordinates of the function $f(z) = u(r, \theta) + iv(r, \theta)$ then show that $\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta} = -r\frac{dv}{dr}$.

18. Find the Analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

19. If $f(z) = u + iv$ is Analytic function then show that

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2.$$

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. Evaluate : $\int_0^{1+i} (x^2 - iy)dz$ along the line $y = x$ and $y = x^2$.

21. State and prove Cauchy's integral formula.

22. Evaluate $\int_C \frac{(z-5)}{(z-3)(z+1)} dz$ where C is the circle $|z| = 2$.

23. Evaluate $\int_C \frac{e^{3z}}{(z+1)^3} dz$ where C is the circle $|z| = 3$.

24. State and prove Morera's theorem.

V. Answer any **FOUR** of the following : **(4 × 5 = 20)**

25. Construct forward difference table for the polynomial $f(x) = x^3 + x^2 - 2x + 1$ for $x = 0(1)5$ and extend the table containing $x = 6$.

26. Given $\sin 45° = 0.7071$, $\sin 50° = 0.7660$, $\sin 55° = 0.8192$, $\sin 60° = 0.8660$, find the value of $\sin 52°$.

27. Find the values of $f(8)$ and $f(12)$ from the following table :

x :	4	5	7	10	11	13
y :	48	100	294	900	1210	2028

28. Find $f'(1)$ and $f''(1)$ from the table :

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

29. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Simpson's 3/8th rule and hence obtain the value of $\log_e 2$.
