Q.P. Code – 22634

Sixth Semester B.Sc. Degree Examination, September 2020

(Semester Scheme)

MATHEMATICS

Paper VIII

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Show that $Amp(z-1)=\frac{\pi}{2}$ represents a line parallel to the imaginary axis.

2. Evaluate $\lim\limits_{z \to -2i} \frac{(2z+3)(z-1)}{(z^2-2z+4)}$.

3. Show that $f(z)=xy+iy$ is not an analytic function.

4. Define :

   (a) Analytic function

   (b) Singular point

5. State C–R equations in polar form for a complex variable function.

6. Evaluate $\int_0^i z^2 dz$ along the line $3y=x$.

7. State Liouville's theorem.

8. Define bilinear transformation.

9. Find the fixed points of the transformation $W=\frac{z-1}{z+1}$.

10. State Fundamental theorem of Algebra.

11. Write the cosine form of the Fourier integral.

12. P.T. $F[f(at)] = \frac{1}{a}\hat{f}\left(\frac{\alpha}{a}\right),\ a > 0$.

13. Find the Fourier cosine transform of e^{-5x}.

14. If a is any non-zero constant and $F_s[f(x)] = \hat{f}_s(\alpha)$,

    then S.T. $F_s[f(x)\sin ax] = \frac{1}{2}\{\hat{f}_e(\alpha - a) - \hat{f}_e(\alpha + a)\}$.

15. Write the formula for inverse k Fourier-cosine transform.

16. Using Bisection method, find a real root of $f(x)$, $x^3 - 4x - 9 = 0$ between 2 and 3 in 3 steps.

17. Explain Newton-Raphson method of solving $f(x) = 0$.

18. Find the largest eigen value of $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ by power method in two steps.

19. Using Euler's method, solve $\frac{dy}{dx} = x + y$, given $y(0) = 1$ for $x = 0.1$.

20. Write the expression to find the constant K_1, K_2, K_3 and K_4 of Runge-Kutta method.

II. Answer any **FOUR** of the following : **(4 × 5 = 20)**

1. S.T. $\arg\left[\frac{z-1}{z+3}\right] = \frac{\pi}{3}$ represents a circle, find its centre and radius.

2. Derive Cauchy-Riemann equation in Cartesian form.

3. Find the analytic function $f(z)$ whose imaginary part is $e^x[x\sin y + y\cos y]$.

4. S.T. $u = 2xy - 2x - 4y$ is harmonic, find the harmonic conjugate.

5. State and prove Cauchy's integral theorem.

6. $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}\,dz$ where C is the circle $|z| = 1.5$.

III. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Discuss the transformation $W = z^2$.

2. S.T. the transformation $W = \frac{1}{z}$ transforms a circle to circle or to a straight line.

3. Find the bilinear transformation which maps the points $z = i, 0, i$ onto $W = -1, i, 1$.

4. S.T. the transformation $W = \frac{4z-5}{2-4z}$ maps the circle $|z| = 1$ onto a circle of radius unity in the W-plane.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the Fourier integral of $f(x) = \begin{cases} 0, & x \le 0 \\ e^{-x}, & x > 0 \end{cases}$.

2. S.T. the Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases} \text{ is } \frac{4}{\sqrt{2\pi}}\left[\frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}\right].$$

3. Find the inverse Fourier transform of $e^{-\alpha^2}$.

4. Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}$$

5. Assume $\frac{1}{\sqrt{x}}$ is self reciprocal w.r.t. Fourier sine transform, deduce that

$$F_S\left[\frac{\cos ax}{\sqrt{x}}\right] = \frac{1}{2}\left[\frac{1}{\sqrt{\alpha + a}} + \frac{1}{\sqrt{\alpha - a}}\right]$$

V. Answer any **THREE** of the following : (3 × 5 = 15)

1. Find the real root of the equation $x^3 - 5x + 3 = 0$ lying between 0 and 1 correct to 3 decimal places by Bisection method.

2. Find a real root of the equation on $x^3 - 5x - 7 = 0$ correct to 3 decimal places using Regula Falsi method.

3. Solve :

$$\begin{aligned} 10x + 2y + z &= 9 \\ x + 10y - z &= -22 \\ -2x + 3y + 10z &= 22 \end{aligned}$$

by Jacobi - iteration method.

4. Using Euler's modified method, solve $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ for $x = 0$ taking $h = 0.1$.

5. Apply Runge-Kutta method to solve

$\frac{dy}{dx} = 3x + \frac{y}{2}$ where $y(0) = 1$, compute $y(0.2)$ by taking $h = 0.2$.